Engineering Notes

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Stability Limits for Downsprings

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Nomenclature

A = constant
C_D = drag coefficient
$C_{H(\alpha)}$ = floating tendency of the elevator
$C_{H(\delta)}$ = restoring tendency of the elevator
C_L = lift coefficient
$C_{L(\alpha)}$ = lift-curve slope
$C_{m(q)}$ = pitch damping $[C_{m(q)} = \partial C_m/\partial q^*,\ q^* = q\bar{c}/(2V)]$
$C_{m(V)}$ = $V\partial C_m/\partial V$ speed dependent pitching moment
$C_{m(\alpha)}$ = angle-of-attack stability
$C_{m(\dot{\alpha})}$ = angle-of-attack damping $[C_{m(\dot{\alpha})} = \partial C_m/\partial \dot{\alpha}^*,\ \dot{\alpha}^* = \dot{\alpha}\bar{c}/(2V)]$
F_d = force exerted on the stick by a downspring
h = altitude
N_o = neutral point
$N_m - \bar{x}_{cg}$ = maneuver margin
K = $\partial C_D/\partial(C_L^2)$ drag due to lift
k_y = square of the ratio of the radius of gyration and the mean aerodynamic chord
n_V = denoting thrust change ΔT due to speed change ΔV, $\Delta T/T = n_V\,\Delta V/V$
μ = relative density of the airplane, $\mu = m/(\rho S\bar{c})$
τ = relative control effectiveness of the elevator
$'$ = control-free values, e.g., $C'_{m(\alpha)}$

DOWNSPRINGS are often used to improve control-free static stability and control force gradients in case of unpowered control systems.[1,2] However, they also can cause undesirable effects which degrade the dynamic behavior of the airplane. In particular, the stability boundaries are of interest, since too large a downspring makes the airplane dynamically unstable.[2-5] The purpose of this Note is to give a more detailed insight into the stability characteristics and to show the conditions most restrictive, with emphasis placed on light airplanes.

In many cases, the elevator mode is well separated from the airplane's modes of motion. The effect of freeing the control can then be investigated by modifying the stability derivatives according to the floating and restoring tendency of the elevator. As a consequence, the analysis is based on a stability quartic similar to the control-fixed case, with the boundaries of the stability region determined by the constant term of the characteristic equation (divergence boundary) and by Routh's discriminant (oscillatory boundary).[1-3] Applying a downspring results in a speed-dependent pitching moment

$$C'_{m_V} = A\,C_L F_d \tag{1}$$

where F_d is the force exerted by the downspring on the stick and A represents a constant given by the characteristics of the airplane.[6] With the use of $C'_{m(V)}$, the divergence boundary (static stability boundary) may be written as

$$C'_{m_V}/C_L \approx 2\,C'_{m_\alpha}/C'_{L_\alpha} \tag{2}$$

The main properties of the stability characteristics, as illustrated in Fig. 1 for a single-engine light airplane, can be shown to be as follows.[6] The point of intersection of the divergence and oscillatory boundaries varies little with C_L and it is approximately given by

$$\begin{aligned} C'_{m_\alpha}\big|_I &\approx -C'_{L_\alpha}C'_{m_q}/4\mu \\ C'_{m_V}\big|_I/C_L &\approx -C'_{m_q}/(2\mu) \end{aligned} \tag{3}$$

This point determines the most aft c.g. position up to which an airplane can be stabilized statically and dynamically by downsprings. It represents the control-free maneuver point[3] as well as the neutral point, with the latter moved back from the downspring according to $C'_{m(V|I)}$

Starting from this fixed point in the graph of Fig. 1, the region denoted by I with its c.g. range

$$\Delta\bar{x}_I \approx \frac{k_y/\mu}{2-n_V}\,\frac{C_L^2}{C_D} \tag{4}$$

is of particular significance. In this region, the $C'_{m(V)}$-values of the oscillatory boundary are smaller than $C'_{m(V|I)}$ despite the fact that the static and maneuver margins have increased. In particular, the minimum

$$\frac{C'_{m_V}\big|_{\min}}{C'_{m_V}\big|_I} \approx \frac{C_D/C_L^2 - (C_D/C_L^2)_{\rm crit}}{C_D/C_L^2 - 2K(C_D/C_L^2)_{\rm crit}/(K - k_y/C'_{m_q})} \tag{5}$$

together with the corresponding c.g. position

$$\frac{\Delta\bar{x}_{\min}}{\Delta\bar{x}_I} \approx \frac{1}{2} - \frac{K}{k_y}\left(\mu\frac{C'_{m_V}\big|_{\min}}{C_L} + \frac{C'_{m_q}}{2}\right) \tag{6}$$

is of interest since it has to be considered as limiting factor in regard to the downspring size which can be applied in this c.g. range. With a downspring fixed by $C'_{m(V/I)|\min}$, the most aft c.g. position usable is restricted to a point which is

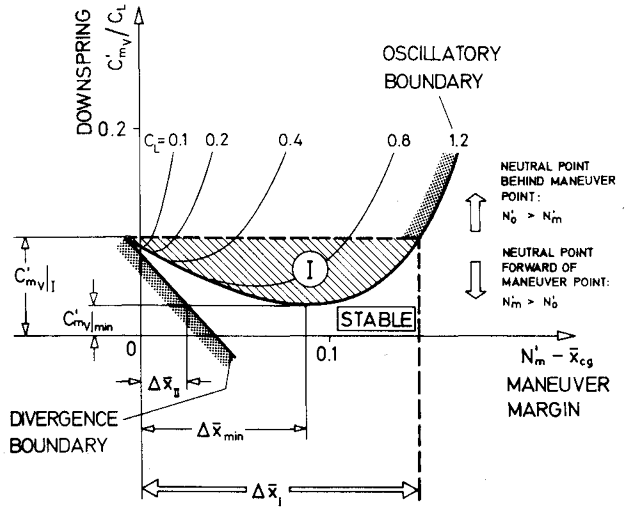

Fig. 1 Stability characteristics of downsprings.

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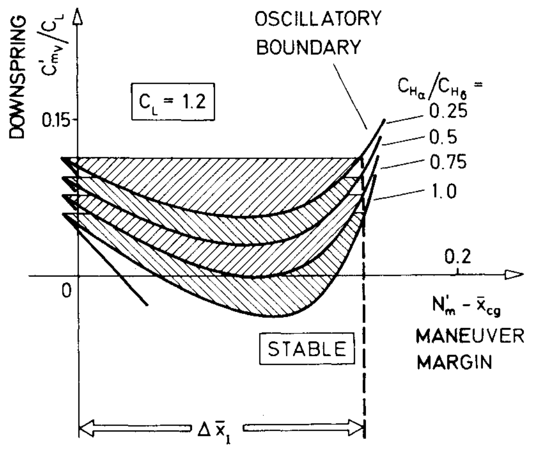

Fig. 2 Effect of the elevator aerodynamics on the oscillatory boundary.

$$\Delta\bar{x}_{II} \approx (0.5/C_L)(C'_{m_V}|_I - C'_{m_V}|_{\min}) \tag{7}$$

forward of the maneuver point as the utmost limit possible. As shown in Fig. 1, the decisive quantity in this context is the lift coefficient C_L which enlarges the region I both by increasing the c.g. range $\Delta\bar{x}_I$ and by reducing $C'_{m(V)}|_{\min}$. Consequently, the $C'_{m(V)}$-values causing dynamic instability may be much smaller than $C'_{m(V),I}$, up to which the airplane is usually considered to be stable.[3]

If $C_L{}^2/C_D$ is equal to the critical value

$$\left(\frac{C_L{}^2}{C_D}\right)_{\text{crit}} \approx (2 - n_V)\frac{C'_{m_q}}{C'_{L_\alpha}}\,\frac{C'_{m_q} + C'_{m_{\dot\alpha}} - 2k_y C'_{L_\alpha}}{2(k_y - K\,C'_{m_q})^2} \tag{8}$$

the oscillatory boundary even reaches the N'_m-$\bar{x}_{cg}$-axis, in a distance

$$\frac{\Delta\bar{x}_{\text{crit}}}{\Delta\bar{x}_I} \approx \frac{1}{2} - \frac{K\,C'_{m_q}}{2k_y} \tag{9}$$

forward of the maneuver point. This condition is shown in Fig. 2. As a consequence, a downspring applied in such a case increases the dynamic instability already existing, despite the fact that the static margin is positive and the neutral point is forward of the maneuver point.

This condition is based on a combination of high C_L and light pitch damping, from which the latter is varied in Fig. 2 with the use of the aerodynamic characteristics of the elevator according to

$$\begin{aligned} C'_{m_q} &\approx (1 - \tau C_{H_\alpha}/C_{H_\delta})C_{m_q} \\ C'_{m_{\dot\alpha}} &\approx (1 - \tau C_{H_\alpha}/C_{H_\delta})C_{m_{\dot\alpha}} \end{aligned} \tag{10}$$

It is interesting to note that the reduction of $C'_{m(q)}$ and $C'_{m\dot\alpha}$, which considerably decreases the minimum $C'_{m(V)}|_{\min}$, does not alter the c.g. range $\Delta\bar{x}_I$, as shown by Eq. (4).

The effect presented in Fig. 2 is also of interest in regard to the design of the elevator aerodynamics and their influence on static and dynamic stability. As to the static conditions, the reductions of the static and maneuver margins caused by an increase of the ratio $C_{H(\alpha)}/C_{H(\delta)}$ can always be compensated by an appropriate application of downsprings and bobweights. However, the demand for larger downsprings and bobweights in such a case is combined with an increased susceptibility to dynamic instability, particularly in the c.g. range $\Delta\bar{x}_I$.

When the altitude is varied, the density change may cause further restrictions. In order to show this effect it is appropriate to alter the scales of the axes by the factor μ $= m/(\rho S\bar{c})$ as done in the diagram in the upper half of Fig. 3. This eliminates the density of the air in regard to the curves describing the stability boundaries. Thus, they are valid for all altitudes. This is based on the fact that $C'_{m(\alpha)}$ and $C'_{m(V)}$ are the only quantities of importance which are combined with μ in the equations of the stability boundaries. The only quantity changing with μ in this diagram is a given downspring according to Eq. (1). It moves on a straight line going through the point $N'_{o,\text{basic}}$ (neutral point in case of $C'_{m(V)} = 0$), with the distances being reciprocal to the density. As a consequence, the stability region is divided into three parts A, B, and C by the tangent going from $N'_{o,\text{basic}}$ to the most restrictive oscillatory boundary. There is always stability in Pt. A. In B and C, however, restrictions due to altitude changes have to be considered. According to the shift of a given downspring in proportion to μ, the most restrictive case for part B is given by low altitude flight. Contrary to this, the downspring size applicable in Pt. C is decreased as the altitude is increased. This is illustrated in the lower half of Fig. 3 where the effect of altitude on the low-frequency roots at various lift coefficients is shown. The downspring characteristics which the airplane is assumed to have are chosen such as to illustrate the different behavior in Pts. B and C.

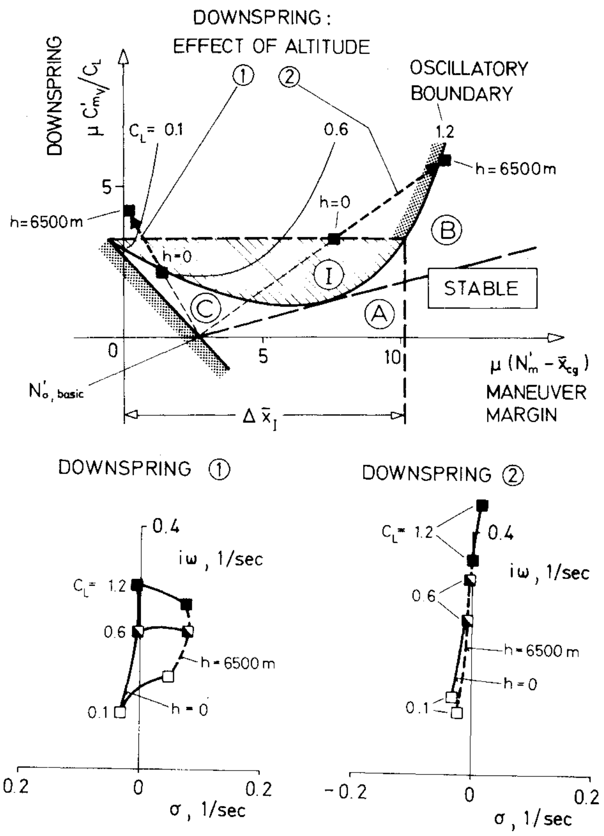

Fig. 3 Effect of altitude and lift coefficient on the stability characteristics.

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Potential Flow Past Annular Aerofoils

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Nomenclature

F_p = prescribed normal velocity boundary condition
G = strength of mass flow control vortex
K = strength of Kutta vortex
N = number of panels used to approximate an annular aerofoil surface
$\vec{n}$ = unit normal vector to a surface or panel
P = a general field point in space or a point on the boundary surface
q = a source point on the body surface
Q = surface source density
V = velocity
V_{nij} = normal component of the induced velocity at the control point of ith surface panel by a unit value of source density of the jth surface panel
V_{tij} = tangential component of the induced velocity at the control point of the ith surface panel by a unit value of source density on the jth surface panel.
θ = surface slope of the aerofoil

Subscripts

i = Denotes quantities associated with the ith panel
j = Denotes quantities associated with the jth panel
p = Denotes quantities associated with the boundary
∞ = Denotes quantities associated with the free-stream

THE problem of the subsonic potential flow past annular aerofoils has been treated extensively, recently using the method of singularities.[1,6] Smith and Hess[1] solved the problem by using surface sources and computing the flow properties from the resulting integral equation using high-speed digital computers. Geissler[3] has recently solved the thick annular aerofoil problem using a combination of source and vortex rings. An interesting experimental technique based on the linear theory using rheoelectric analogy[4] has been used for both the direct and inverse problems.

The purpose of this Note is to study the lowspeed potential flow past annular aerofoils using three different methods. The Kutta condition is satisfied at the trailing edge and the mass flow through the body is specified independently. The first is the Young's nonlinear method[2] using a combination of source and vortex distributions. The second is the rheoelectric analogy in which the flow potential is simulated in an inclined tank using the electrical analogy. The third is the Smith's method wherein source distributions are put on the body. All these techniques are applied to NACA-66-006 profile and the relative merits of the three assessed.

Analysis

The basic integral equation for a continuous source distribution on any arbitrary body is given by

$$2\pi Q(P) - \int_S\int \frac{\partial}{\partial n}\left(\frac{1}{r(p,q)}\right) Q(q)dS = -\vec{n}\cdot\vec{V}_\infty + F_p \qquad (1)$$

As explained in Ref. 1, the annular aerofoil is approximated by a large number of panels in the form of conical frustra. The continuous distribution of sources over the body surfaces is replaced by N discrete values, each constant over a segment. The mid point of each segment is chosen as the control point. A surface source strength of unit strength is placed on each element and the normal velocity component to the body surface induced at every control point by all other elements is calculated by numerical integration.

The mass flow through the annular aerofoil can be changed by the addition of a uniform vortex distribution placed on the camber surface of the aerofoil and extending downstream of the trailing edge on the mean cylinder. The strength of this vortex can be varied to give the required intake flow and this induces a normal velocity component at the control points on the surface of the ring aerofoil.

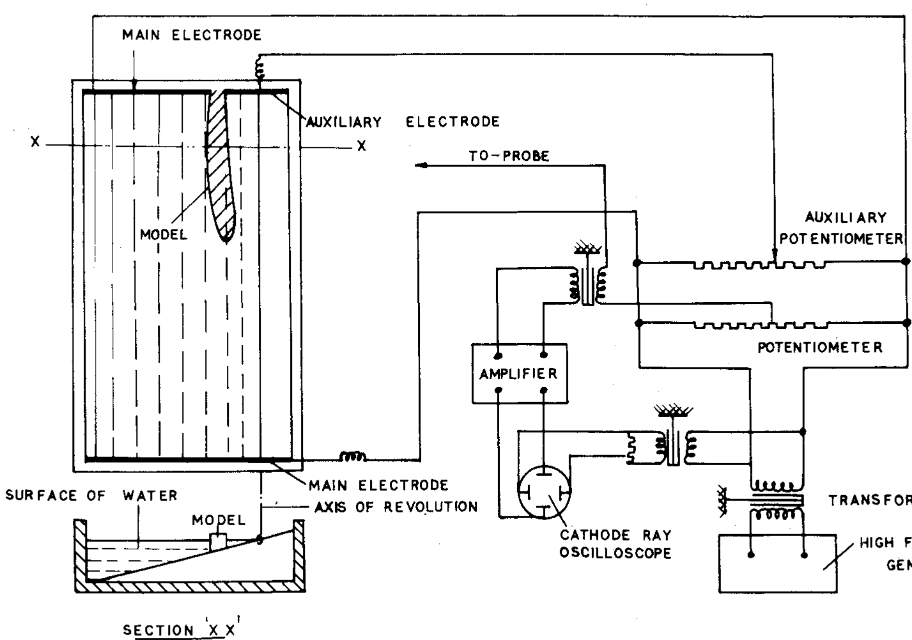

Fig. 1 Rheoelectric analogy-inclined tank, simulation of flow past annular aerofoils with different inlet flow conditions.

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